CLASS EXERCISES

Find the center and radius of each circle whose equation is given.

1. $x^2 + y^2 = 16$
2. $(x-2)^2 + (y-7)^2 = 36$
3. $(x-4)^2 + (y+7)^2 = 7$
4. $x^2 + y^2 + 12y = 0$
5. $x^2 - 2x + y^2 - 6y = 9$
6. $4x^2 + 4y^2 = 36$

In Exercises 7–9, find an equation of the circle described.

7. The circle with center $(7, 3)$ and radius 6.
8. The circle with center $(-5, 4)$ and radius $\sqrt{2}$.
9. The circle with center $(0, 0)$ that passes through $(-5, 12)$.
10. The graph of $x^2 + y^2 = 25$ consists of all points in the plane that are 5 units from the origin. Describe the graphs of:
    a. $x^2 + y^2 < 25$ **b.** $x^2 + y^2 > 25$
11. ***Discussion*** Suppose that you wish to find where the line $3y + x = 6$ intersects the circle $x^2 + y^2 = 10$. Describe what you would do.
12. ***Discussion*** To find the intersection of the line $y = x + 8$ and the circle $x^2 + y^2 = 16$, Janice solved the equations simultaneously and found that $x = -4 \pm 2i\sqrt{2}$. What do the imaginary roots tell her?
13. ***Discussion*** Describe how to use a graphing calculator to graph $x^2 + y^2 = 9$ so that the circle does not appear distorted or flattened.

WRITTEN EXERCISES

In Exercises 1–12, write an equation of the circle described.

A

1. $C(4, 3)$, $r = 2$
2. $C(5, -6)$, $r = 7$
3. $C(-4, -9)$, $r = 3$
4. $C(a, b)$, $r = f$
5. $C(6, 0)$, $r = \sqrt{15}$
6. $C(-4, 2)$, $r = \sqrt{7}$
7. The center is $(2, 3)$; the circle passes through $(5, 6)$.
8. The points $(8, 0)$ and $(0, 6)$ are endpoints of a diameter.
9. The center is $(5, -4)$ and the circle is tangent to the x-axis.
10. The center is $(-3, 1)$ and the circle is tangent to the line $x = 4$.
11. The circle is tangent to the x-axis at $(4, 0)$ and has y-intercepts -2 and -8.
12. The circle contains $(-2, 16)$ and has x-intercepts -2 and -32.

Write each equation in center-radius form. Give the center and radius.

13. $x^2 + y^2 - 2x - 8y + 16 = 0$
14. $x^2 + y^2 - 4x + 6y + 4 = 0$
15. $x^2 + y^2 - 12y + 25 = 0$
16. $x^2 + y^2 + 14x = 0$
17. $2x^2 + 2y^2 - 10x - 18y = 1$
18. $2x^2 + 2y^2 - 5x + y = 0$

19. Show that the line $y = 2x + 8$ contains the center of the circle $x^2 + y^2 + 6x - 4y + 8 = 0$.
20. Determine whether the line $3x + 2y = 6$ contains the center of the circle $x^2 + y^2 + 4x - 12y + 24 = 0$.

In Exercises 21–23, use a graphing calculator or computer software to graph each equation.

21. $x^2 + y^2 = 50$
22. $(x-3)^2 + y^2 = 36$
23. $x^2 + y^2 - 6y = 40$

In Exercises 24–26, on a single set of axes, sketch the graph of each semicircle whose equation is given.

24. **a.** $y = \sqrt{9 - x^2}$ **b.** $y = -\sqrt{9 - x^2}$
25. **a.** $x = \sqrt{9 - y^2}$ **b.** $x = -\sqrt{9 - y^2}$
26. **a.** $y = \sqrt{16 - (x-5)^2}$ **b.** $x = 5 - \sqrt{16 - y^2}$

In Exercises 27–34, graph the equations. Solve the equations simultaneously to find the coordinates of any intersection points of their graphs. If the graphs are tangent or fail to intersect, say so.

27. $x + y = 23$, $x^2 + y^2 = 289$
28. $9y - 8x = 10$, $x^2 + y^2 = 100$
29. $2x - y = 7$, $x^2 + y^2 = 7$
30. $x + 2y = 10$, $x^2 + y^2 = 20$
31. $5x + 2y = -1$, $x^2 + y^2 = 169$
32. $y = 5$, $x^2 + y^2 - 4x - 6y = -9$
33. $y = \sqrt{3}x$, $x^2 + (y-4)^2 = 16$
34. $x - y = 3$, $x^2 + y^2 - 10x + 4y = -13$
35. ***Writing*** Write a description of the graphs of $x^2 + y^2 < 1$ and $x^2 + y^2 > 1$.
36. Sketch the graph of $(x-3)^2 + (y-4)^2 \leq 25$.

B

37. The line $x - 2y = 15$ intersects the circle $x^2 + y^2 = 50$ in points A and B. Show that the line joining the center of the circle to the midpoint of $\overline{AB}$ is perpendicular to $\overline{AB}$.
38. Find the length of a tangent line segment from $(10, 5)$ to the circle $x^2 + y^2 = 25$.
39. $P(2, 3)$ is on the circle with center $O(0, 0)$.
    a. Write an equation of the circle.
    b. Write an equation for the tangent l to the circle at P. (*Hint:* l is perpendicular to $\overline{OP}$.)
40. Show that $P(4, 2)$ is on the circle with equation $(x-3)^2 + (y-4)^2 = 5$. Find an equation of the tangent to the circle at P. (*Hint:* See the hint for Exercise 39.)
41. A circle with center $C(2, 4)$ has radius 13.
    a. Verify that $A(14, 9)$ and $B(7, 16)$ are points on this circle.
    b. If M is the midpoint of $\overline{AB}$, show that $\overline{CM} \perp \overline{AB}$.