42. A circle with center $C(-4, 0)$ has radius 15.

a. Verify that $A(8, 9)$ and $B(-13, 12)$ are points on this circle.

b. Write an equation of the perpendicular bisector of $\overline{AB}$ and show that the coordinates of point C satisfy the equation.

c. What theorem from geometry does this exercise illustrate?

43. A diameter of a circle has endpoints $A(13, 0)$ and $B(-13, 0)$.

a. Show that $P(-5, 12)$ is a point on this circle.

b. Show that $\overline{PA}$ and $\overline{PB}$ are perpendicular.

44. a. Find the coordinates of A and of B if $\overline{AB}$ is a horizontal diameter of the circle $x^2 + y^2 - 34x = 0$.

b. Show that $P(2, 8)$ is a point on this circle and that $\overline{PA} \perp \overline{PB}$.

45. Given $O(0, 0)$ and $N(12, 0)$, find an equation in terms of x and y for all points $P(x, y)$ such that $\overline{PO} \perp \overline{PN}$. Simplify this equation and show that P is on a circle. What are the center and radius of the circle?

46. ***Discussion*** Given $A(6, 8)$ and $B(-6, -8)$, write an equation in terms of x and y for all points $P(x, y)$ such that $\overline{PA} \perp \overline{PB}$. Simplify the equation and interpret your answer.

You may find it helpful to have a graphing calculator to complete Exercises 47–49.

47. A triangle is inscribed in a semicircle as shown at the right.

a. Find an equation for the semicircle.

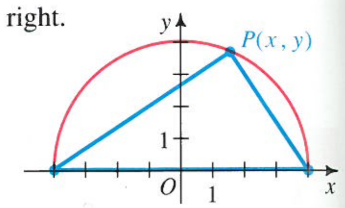
b. Write a function $A(x)$ for the area of the triangle.

c. What is the domain of the function from part (b)?

d. Graph $y = A(x)$.

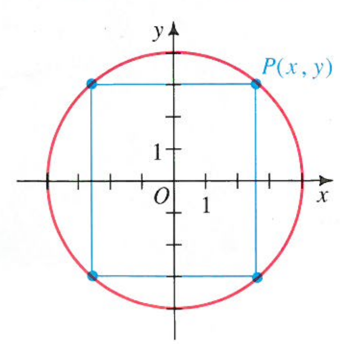
e. Use the graph from part (d) to find the value of x that maximizes $A(x)$.

f. What is the maximum value of $A(x)$?

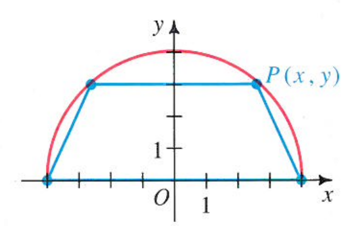


48. a. A rectangle is inscribed in a circle of radius 4 as shown at the left below. Write a function $A(x)$ for the area of the rectangle.

b. Graph $y = A(x)$. Use the graph from part (a) to find the value of x that maximizes $A(x)$. What is the maximum value of $A(x)$?



Ex. 48



Ex. 49

49. a. An isosceles trapezoid is inscribed in a semicircle of radius 4 as shown at the bottom of page 224. Write a function $A(x)$ for the area of the trapezoid.

b. Find the value of x that maximizes $A(x)$.

Find an equation of the circle that contains the given points.

50. $A(0, 0)$, $B(2, 0)$, and $C(2, 2)$

51. $P(0, 0)$, $Q(6, 0)$, and $R(0, 8)$

52. $L(8, 2)$, $M(1, 9)$, and $N(1, 1)$

53. $D(7, 5)$, $E(1, -7)$, and $F(9, -1)$

In Exercises 54–57, describe the set of points satisfying each equation.

54. $x^2 + y^2 + 2x + 2y + 2 = 0$

55. $x^2 + y^2 - 6x + 8y + 26 = 0$

C 56. $(x^2 + y^2 - 1)(x^2 + y^2 - 4) = 0$

57. $x^3y + xy^3 - xy = 0$

58. a. A point (x, y) lies inside the circle $x^2 + y^2 = 2$ and above the line $y = 1$. Give two inequalities that must be satisfied.

b. Sketch the region in which the point lies and find the area of the region.

59. a. Sketch the set of points that satisfies $|x| \geq 1$ and $x^2 + y^2 \leq 4$.

b. Find the area of the region.

60. Suppose that point $P(a, b)$ is any point on the circle with center $O(0, 0)$ and radius r. Suppose that line l is perpendicular to $\overline{OP}$ at P. Prove that l is tangent to the circle as follows:

a. Show that the equation of l can be written $ax + by = r^2$.

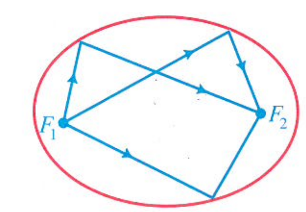
b. Solve the equations of l and the circle simultaneously. Show that there is only one solution.

6-3 Ellipses

Objective *To find equations of ellipses and to graph them.*

If you stand a few feet away from a wall and shine a flashlight against it, you can make a lighted area in the form of an oval, or *ellipse*. The ellipse appears to be an elongated circle. Try it.

Ellipses are found in many applications. Light or sound waves from one *focus* F_1 are reflected from the ellipse to the other focus F_2. This reflection property is used to make a whispering chamber, where a person whispering at F_1 can be heard at F_2. The United States Capitol Building has such a chamber.



To sketch an ellipse, complete the Activity on the next page.